Progress on quantitative modeling of rf sheaths*

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Outline

- **Introduction**
- rf sheath model BC
- Semi-analytic solutions
- Numerical solutions

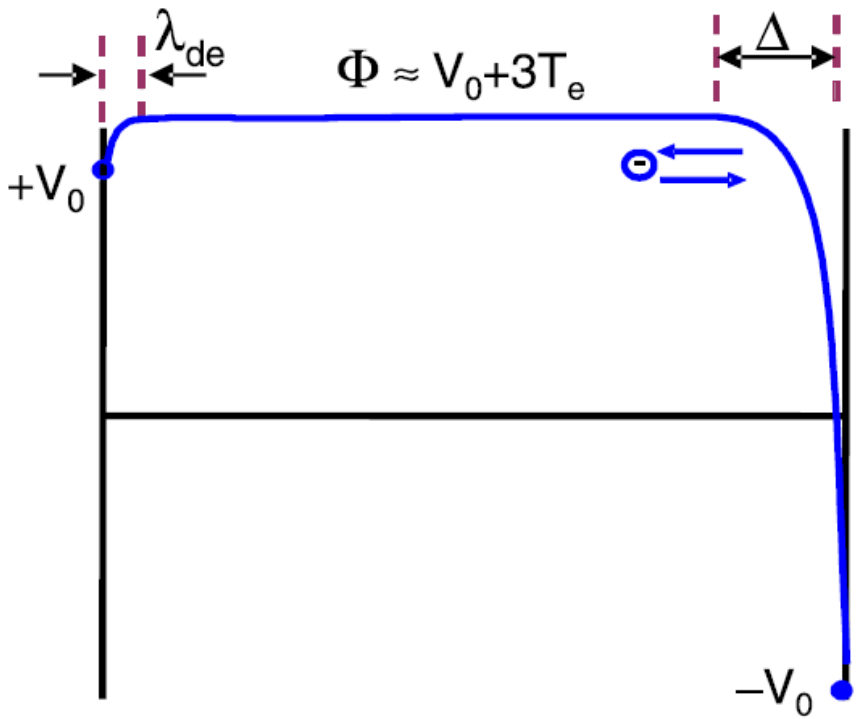
Physics of rf sheaths

- electron acceleration in $E_{\|}$ fields
  $\Rightarrow$ rf sheaths
  $\Rightarrow$ equalize e^- and ion loss rates
  (confine e^-, accelerate ions out)

- geometry is important (B field, antenna and wall)

- B field misalignment with either the antenna or the wall $\Rightarrow$ $E_{\|}$
  $\Rightarrow$ slow wave (SW) $\Rightarrow$ rf sheath

- rf sheaths $\Rightarrow$ rf-enhanced impurities, loss of coupled power, hot spots



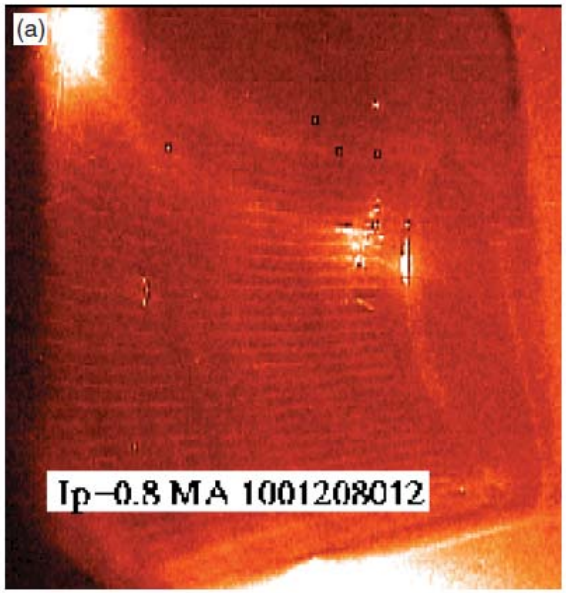
Time-averaged sheath:
Bohm + rectified rf potential

Sheath effects impact rf experiments

- **experiments on JET, TFTR, AUG, C-Mod, NSTX, Tore Supra (and earlier machines) have observed a number of sheath-plasma interactions, e.g.**

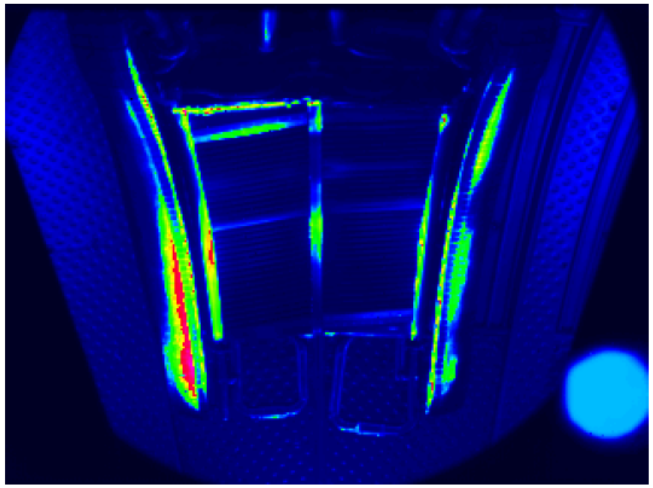
ion acceleration in sheaths ⇒ power dissipation into frame of antenna

⇒ hot spots, reduced heating efficiency



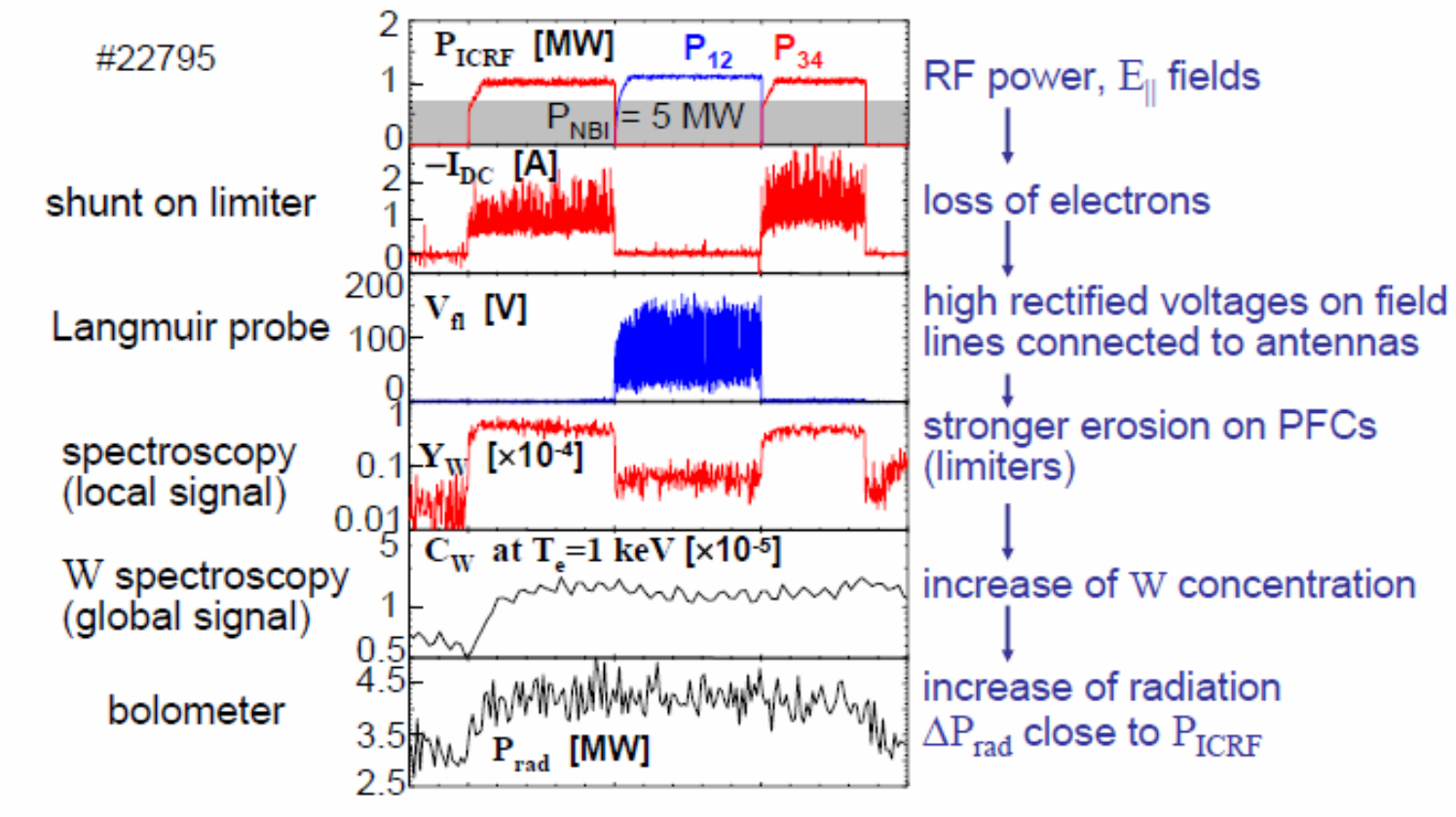
C-Mod (Wukitch, 2010)

Tore Supra
(Colas, 2006)



rf sheaths increase impurities

ASDEX Upgrade with full W (no boronization)



(Bobkov, PSI 2008, NF 2010)

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Small λ_D suggests a BC approach

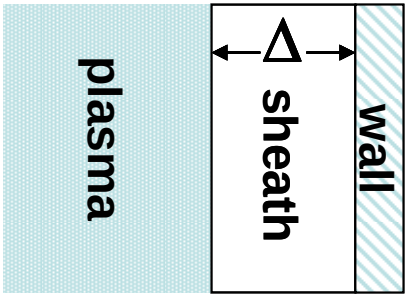
- Early work on rf sheaths estimated the driving sheath potential using an integral of the vacuum SW $E_{\|}$ between B field contact points.

$$V_{sh} = \int ds\ E_{\|}^{(vac)}$$

- The following ordering suggests a different approach:

$$\lambda_D << \delta_e,\ L_{\|}(m_e / m_i)^{1/2} << \delta_i$$

sheath : SW : FW



- **rf sheath BC**: the electron-poor sheath is treated as a thin vacuum layer using
  - total rf field including plasma effects.
  - time-averaged sheath width $\Delta \sim \lambda_D$

BC includes sheath capacitance

- The form of the BC is derived from Maxwell's eqs. using continuity of $\mathbf{E}_t$ and D_n across the plasma-vacuum interface

$$\mathbf{E}_t = \nabla_t (\Delta D_n)$$

Fields defined on plasma side of interface, t = tangential, n = normal

(D'Ippolito and Myra PoP 2006, Myra et al., PoP 1994)

- Sheath capacitance $\propto 1/\Delta$ (sheath impedance $Z_C \propto \Delta \Rightarrow$ RHS term)
- This approach incorporates both plasma effects and detailed geometry in the calculation of the sheath potential.
- This BC depends on both **B**-field and wall geometry: $\parallel$ and $\perp$ to **B**, normal and tangential to boundary
- Sheath BC $\rightarrow$ metal wall BC ($E_t \rightarrow 0$) as $\Delta \rightarrow 0$

Λ is critical parameter for ES sheath

Derivation of sheath capacitance parameter (in ES limit) is

$$\mathbf{E}_t = \nabla_t(\Delta D_n)$$

$$\Rightarrow -\Phi \sim \Delta\,(\mathbf{n}\cdot\mathbf{b}\;\varepsilon_{||}E_{||}) \equiv \mathbf{n}\cdot\mathbf{b}\;\Lambda\;\Phi \qquad \text{where} \qquad \boxed{\Lambda \equiv -ik_{||}\Delta\varepsilon_{||}}$$

For **B** normal to the boundary, there are three limits:

- $|\Lambda| >> 1 \;\Rightarrow$ insulating wall BC ($D_n = 0$)
- $|\Lambda| << 1 \;\Rightarrow$ conducting wall BC ($E_t = 0$)
- $\Lambda = 1 \;\Rightarrow$ sheath-plasma-wave (SPW) resonance when
  - SW is evanescent
  - Z_C (sheath capacitance) balances Z_L (plasma inductance)

Self-consistent rf fields and sheath

sheath BC $\mathbf{E}_t = \nabla_t(\Delta \mathbf{D}_n)$

Child-Langmuir (CL) Law $\Delta = \lambda_D \left(e\Phi_0 / T_e\right)^{3/4}$

sheath potential $V_{sh} = \Delta D_n$, $\Phi_0 = C_{sh} V_{sh}$

rectified sheath potential

- Nonlinear model $\Rightarrow$ self-consistent solution for
  - driving and rectified sheath potentials
  - sheath width
  - rf field at boundary

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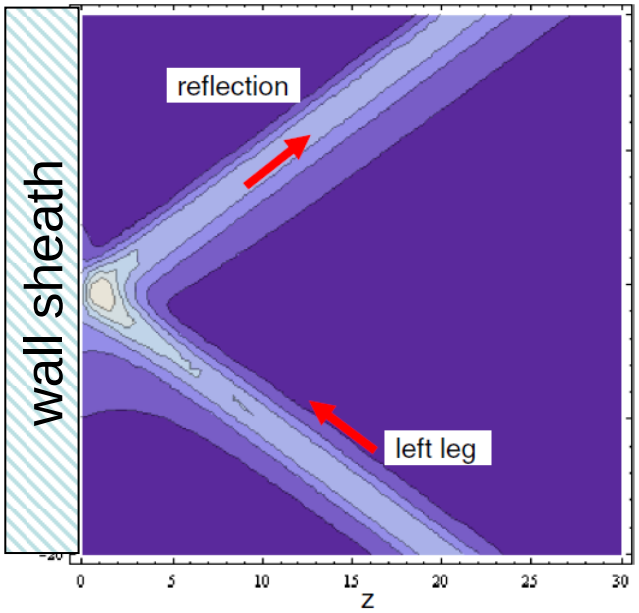
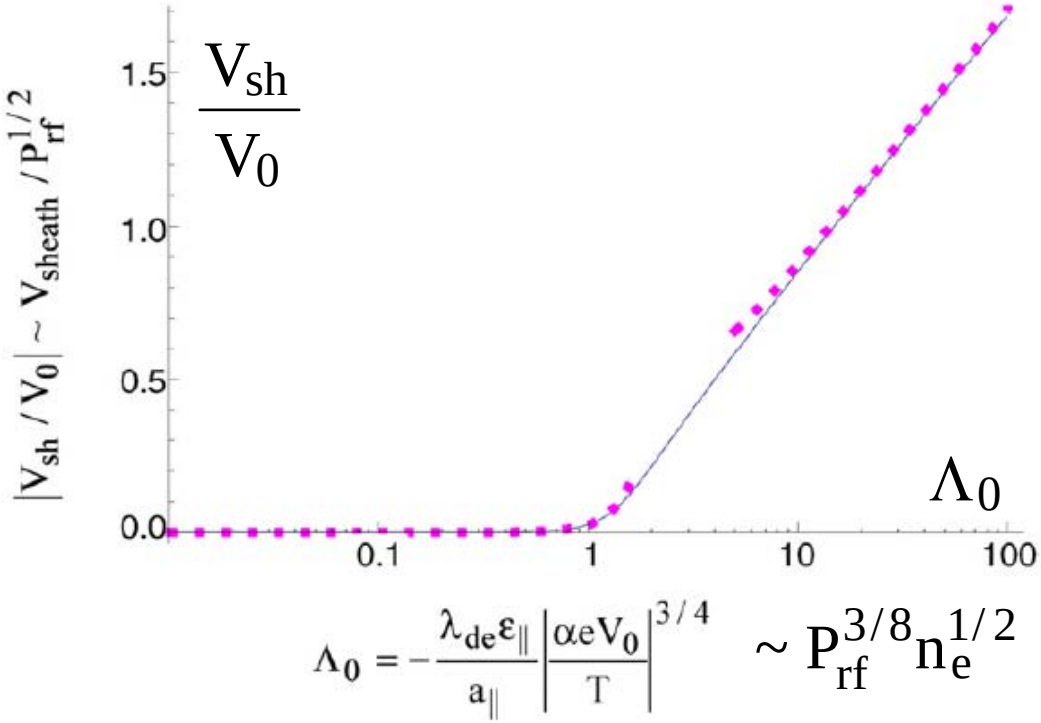
- **Semi-analytic solutions**
- Numerical solutions

$n_e < n_{LH}$: ES SW propagates in resonance cones (RC)

slow wave RC's



(from antenna)



$$\Lambda_0 = -\frac{\lambda_{de}\varepsilon_{\|}}{a_{\|}}\left|\frac{\alpha e V_0}{T}\right|^{3/4} \sim P_{rf}^{3/8} n_e^{1/2}$$

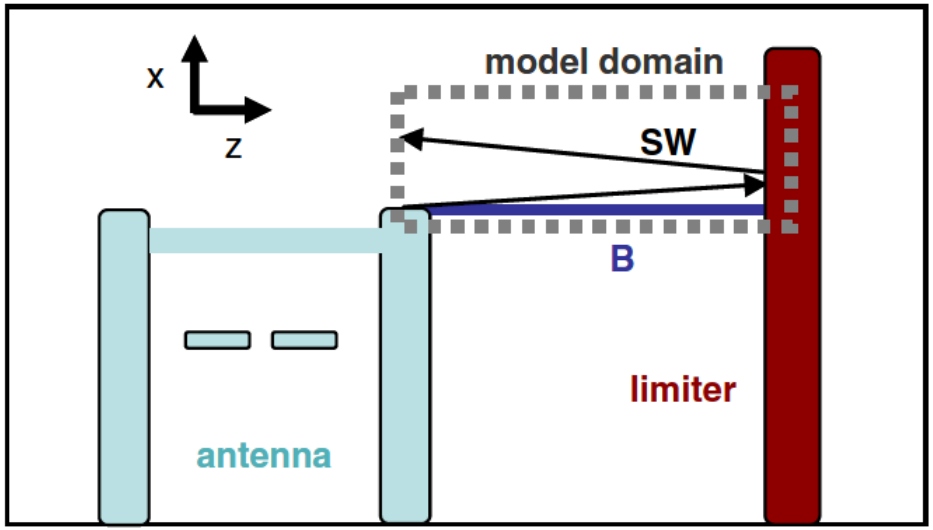
Myra and D'Ippolito, PRL (2008)

(See poster by Ochoukov at this meeting for related C-Mod results)

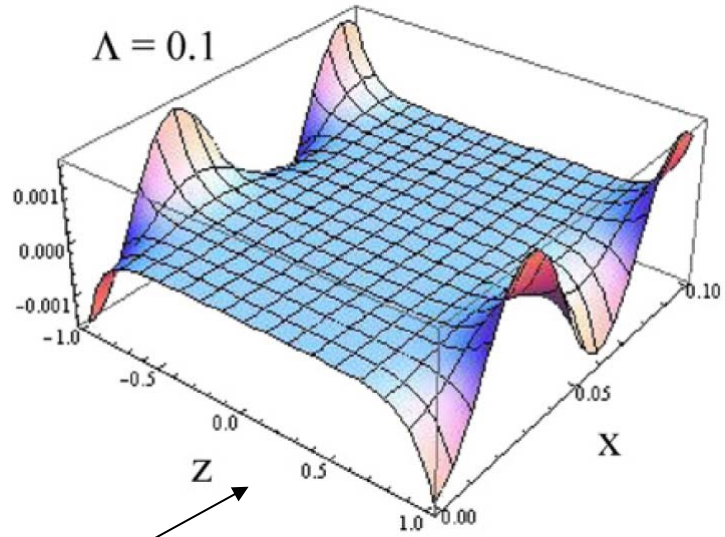
- nonlinear Child-Langmuir constraint

$\Rightarrow$ **voltage threshold** $\Lambda = -\Delta\varepsilon_{\|} / a > 3$ for a significant sheath voltage.

$n_e > n_{LH}$: evanescent SW and propagating sheath-plasma-waves (SPW)



Sketch of the entire problem of interest and the model problem solved here.



SPW localized near sheath and propagating in x

- SW propagates along B and is evanescent in x
- SW couples to a SPW and propagates radially if $L_{||} < \delta_i$ (sufficiently close limiters) and $\Lambda_0 = \Delta\varepsilon_{||}/L_{||} > 1$.

Myra and D'Ippolito, PPCF (2010)

Propagating FW hits wall
⇒ far field sheath with multiple roots

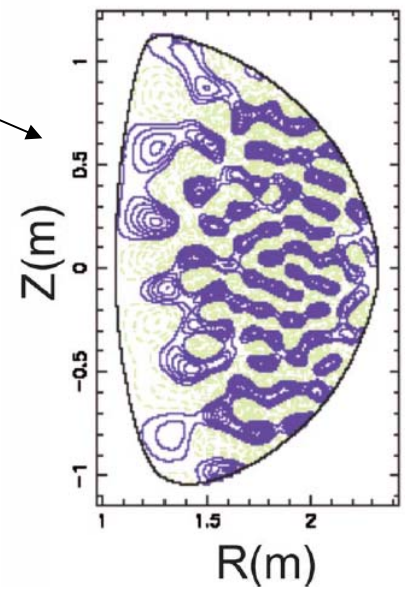
- Poor single pass absorption ⇒ FW fields at wall
- B_0-field has a component normal to wall ($B_x \neq 0$)
- FW cannot satisfy the BC at the far wall without coupling to SW ⇒ "far field sheath"
- e.g. 3-wave solution (index j ⇒ incident FW + reflected FW + SW)

$$E_{SW} = E_{FW} \frac{\mathbf{s}\cdot\mathbf{g}_0 \times \mathbf{g}_1}{\mathbf{s}\cdot\mathbf{g}_1 \times \mathbf{g}_2}$$

D'Ippolito et al, POP (2008)

where $\mathbf{g}_j = \mathbf{e}_j - i\Delta\left(\mathbf{s}\cdot\boldsymbol{\varepsilon}\cdot\mathbf{e}_j\right)\mathbf{k}_j$

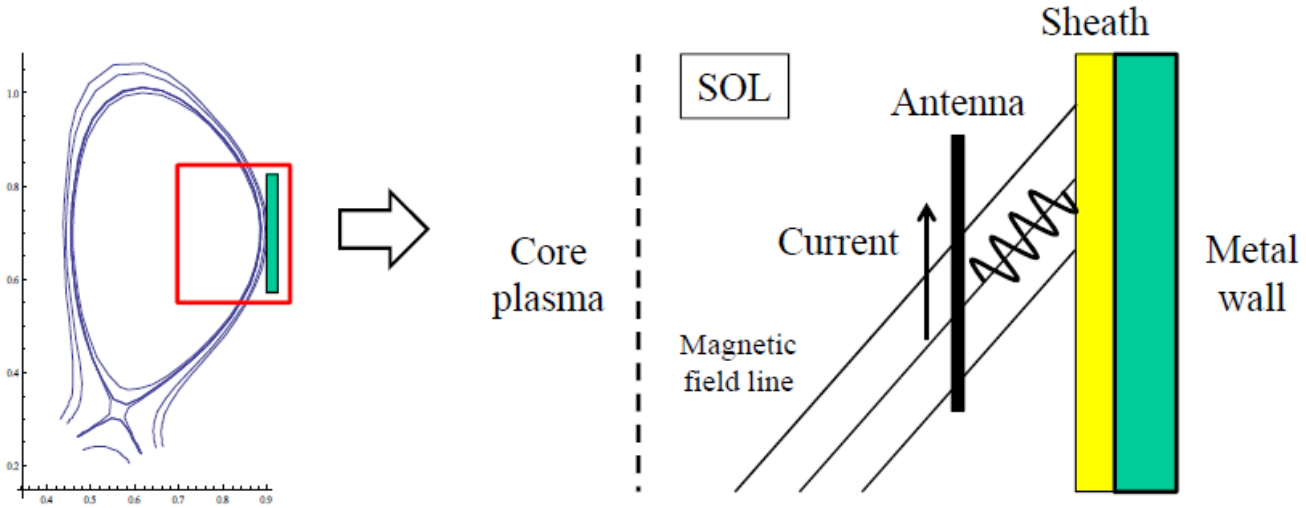
- Sheath BC permits multiple roots and SPW resonance (a numerical example will be discussed subsequently)



2D AORSA (Jaeger)

Outline

- Introduction
- rf sheath model BC
- Semi-analytic solutions
- **Numerical solutions – rfSOL code:**
  "<u>rf</u> waves and sheaths in <u>SOL</u>"



rfSOL code project

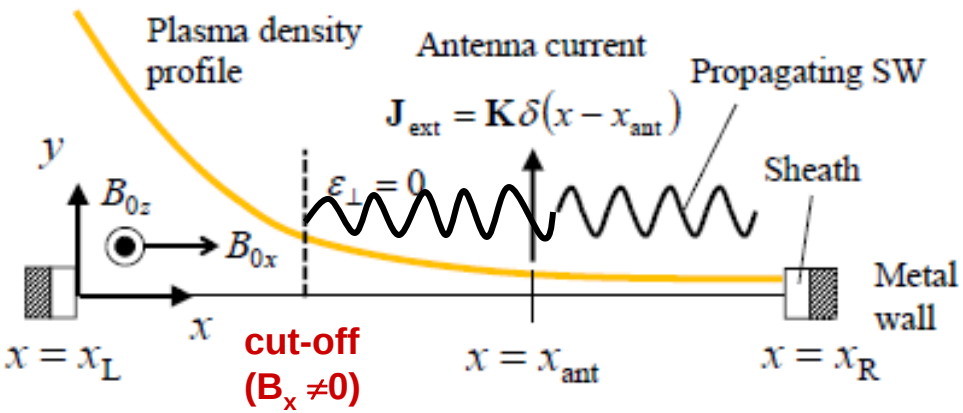
- a new code (rfSOL by H. Kohno) was developed to calculate rf sheath potentials incorporating plasma effects and geometry
- uses Finite Elements method (FEM) allow treatment of arbitrarily shaped boundaries
- code was parallelized to allow greater resolution
- compared code solutions with semi-analytic solutions for 1D and 2D model problems.

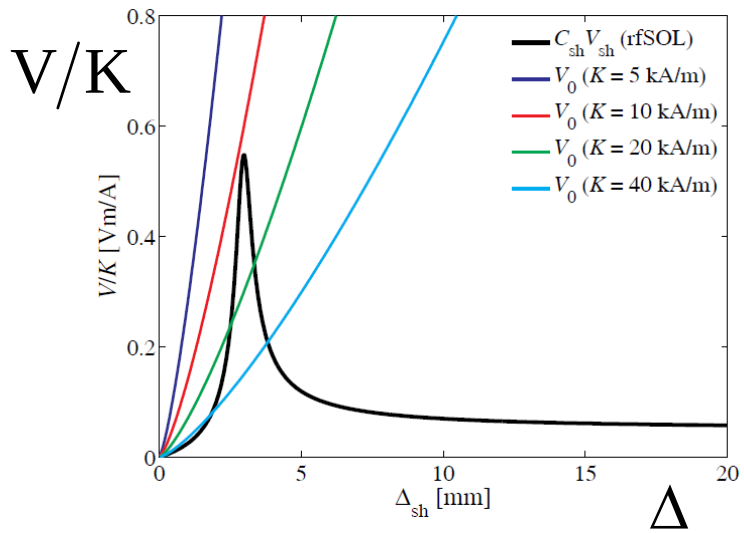
rfSOL recovers physics in analytic models but generalizes them to include 2D, density profiles, model antennas, spatially varying B fields, etc.

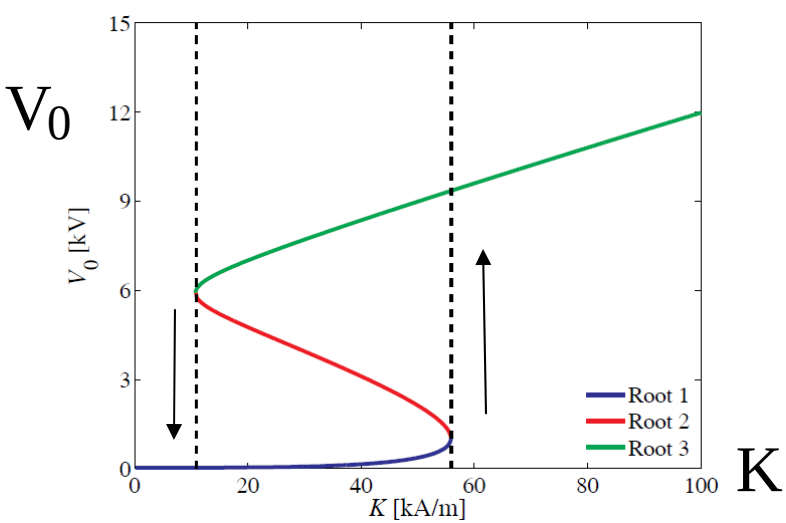
Kohno, thesis (2011)

1D propagating SW ⇒ sheath potential with multiple roots, hysteresis

1D low density ($n_e < n_{LH}$) ⇒ propagating SW





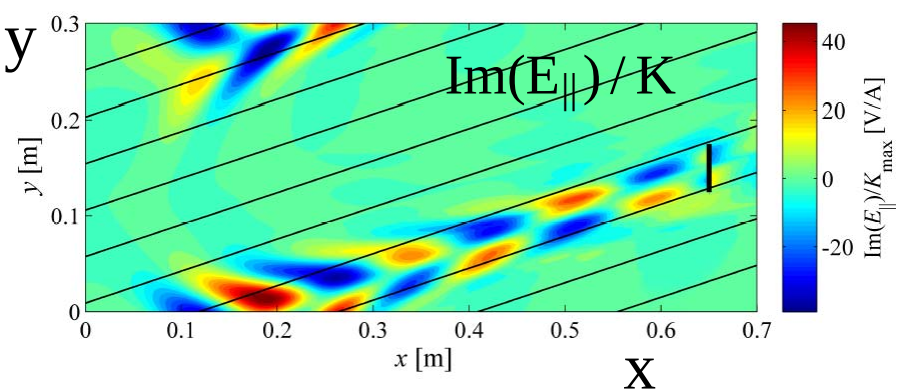


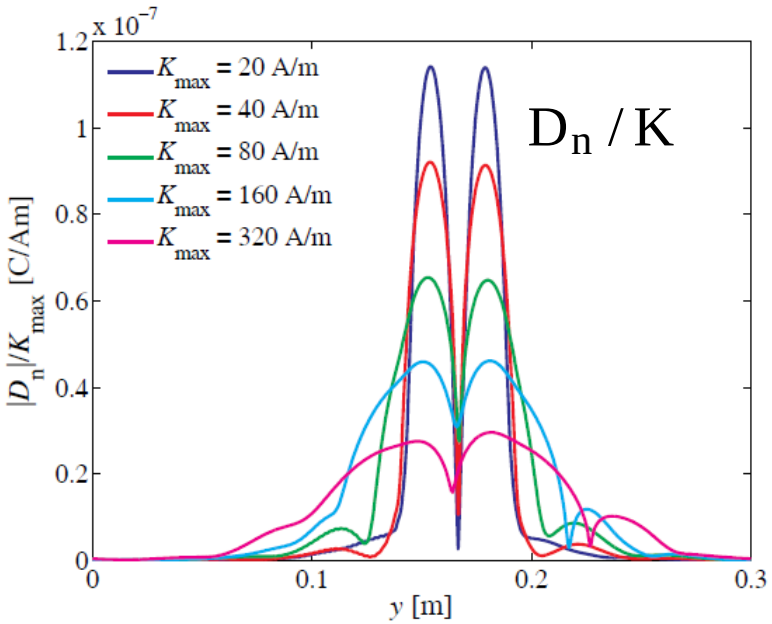
- linear standing wave resonance (dissipation limits amplitude)
- multiple roots due to sheath nonlinearity (CL Law)
- root merging ⇒ hysteresis

2D propagating SW ⇒ resonance cones

2D low density ($n_e < n_{LH}$) ⇒

propagating SW resonance cones





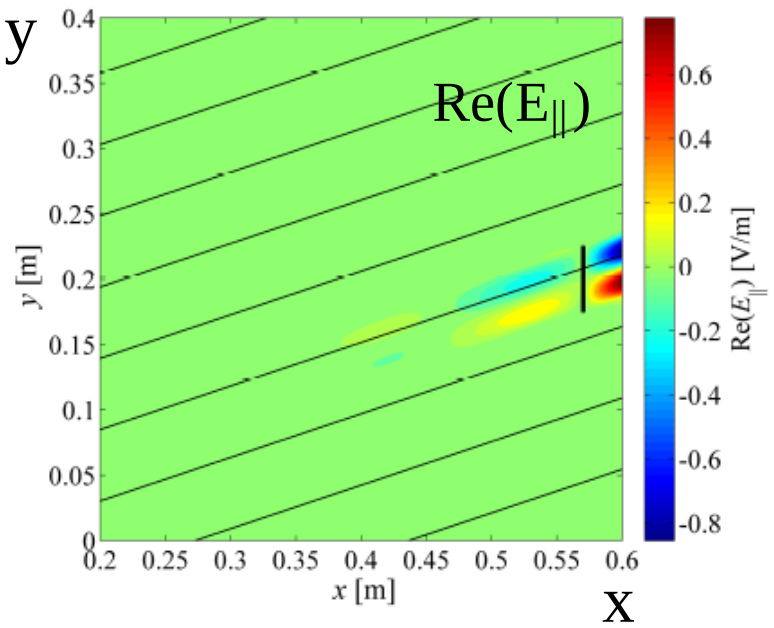
• self-consistent nonlinear sheath ⇒ sheath width Δ ∝ antenna current K

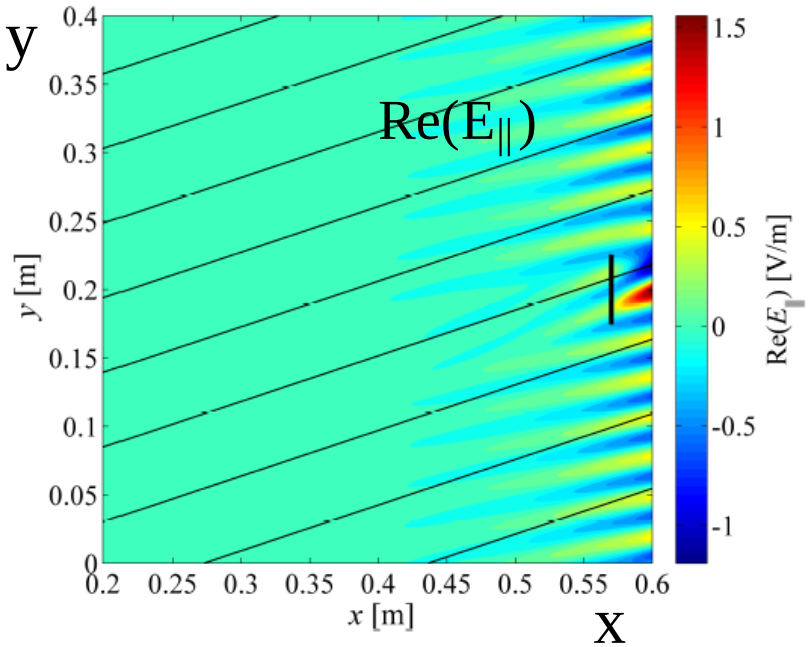
• Sheath BC affects phase shift of wave reflected off wall ⇒ could affect near field of antenna

• insulating BC limit is approached as antenna current K increases (sheath BC ⇒ Δ (D_n/K) ≈ const.)

2D evanescent SW + sheath BC ⇒ propagating sheath-plasma wave (SPW)

2D tilted-B, high density ($n_e > n_{LH}$) ⇒ evanescent SW





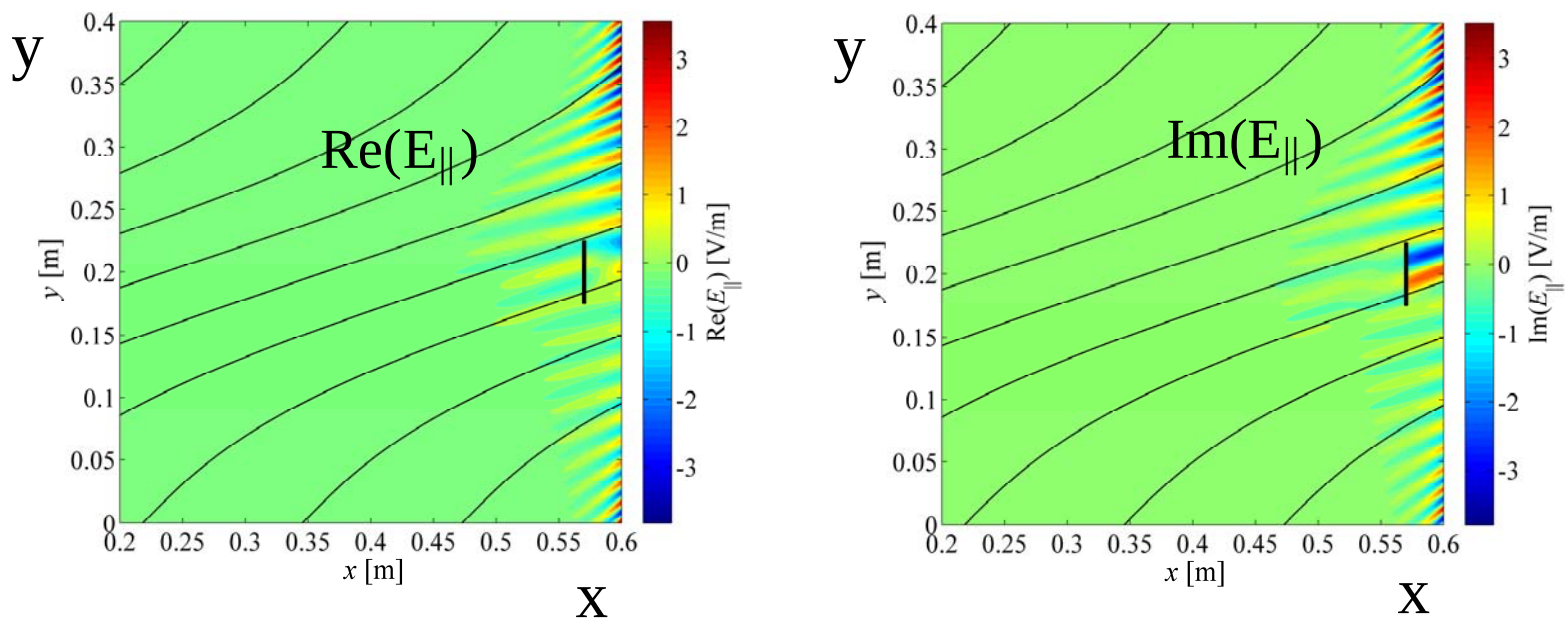
metal wall BC ⇒ no SPW

sheath BC ⇒ SPW

(SPW propagates along sheath, and is localized in x)

SPW is sensitive to B-field geometry

2D *curved-B*, high density ($n_e > n_{LH}$) $\Rightarrow$ evanescent SW



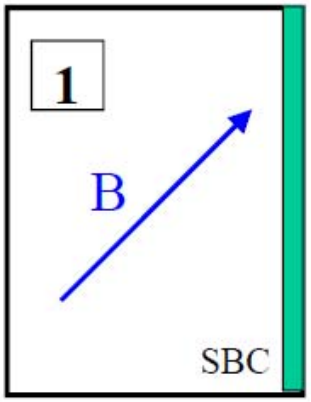
- **wave phase follows field line (couples k_x and k_y)**
- **SPW propagates along sheath $\Rightarrow$ voltage spreading**
- **note the poloidal asymmetry**

Other cases $\Rightarrow$ sheath singularity where $B_n = 0$

Hierarchy of rfSOL models

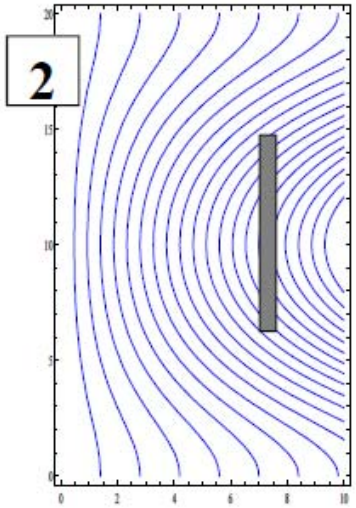
1D / 2D slab

Analytic,
code dev
V&V



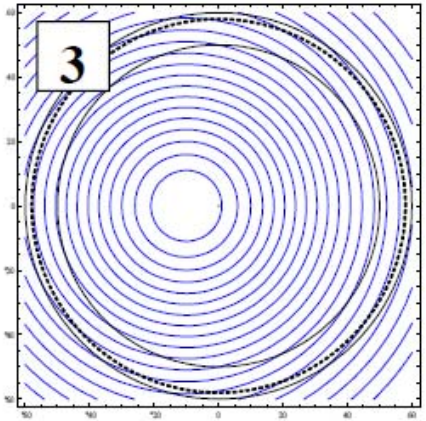
2D slab, varying B

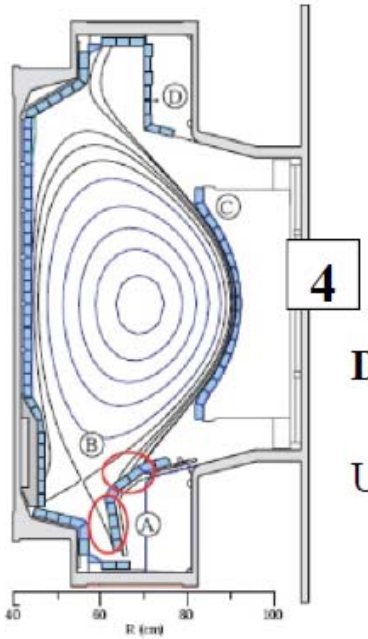
Periodic flux surfaces
Sheath-sheath connections



2D cylindrical

Shifted circle flux surfaces
More realistic connections
FEM technology





Divertor geometry

Ultimate goal

Summary and Future Work

- Unlike previous methods, the rf sheath BC determines the rf sheath potential including
  - plasma dielectric effects
  - nonlinear Child-Langmuir constraint
  - arbitrary geometry of B field and boundary

- Both analytic and numerical solutions have demonstrated $E_{||}$-driven rf sheath formation, sheath plasma waves, multiple roots / hysteresis, and sheath voltage threshold.

- A new FEM code (rfSOL) provides a path forward for calculating rf sheath effects for realistic boundaries in the tokamak SOL.

Extra slides

Origin of poloidal asymmetry for curved B field

- ES sheath BC is given by

$$1 = ik_{\|}\Delta\varepsilon_{\|}$$

$$\Rightarrow k_{\|} = k_x\, b_x + k_y\, b_y \text{ is imaginary}$$

$$\Rightarrow k_x \text{ and } k_y \text{ have imaginary parts}$$

- Note that k_x and k_y are coupled when $b_x \equiv B_x/B \neq 0$.
- Directionality / evanescence in x

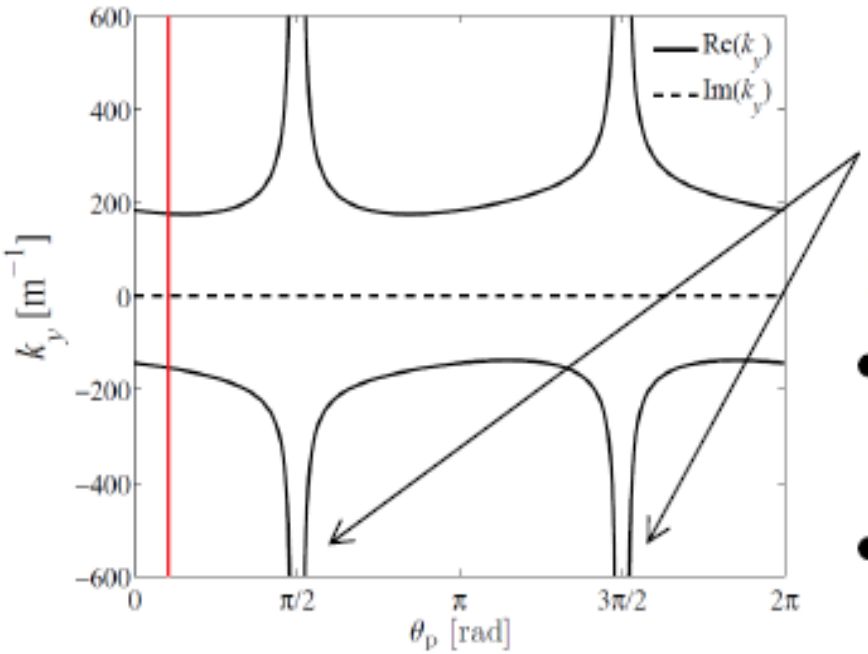
  $\Rightarrow$ directionality / evanescence in y.

2D singularity problem

$$\mathbf{E}_t = \nabla_t(\Delta D_n) \approx i\,k_t \Delta\,(\mathbf{n}\cdot\mathbf{b}\;\varepsilon_{||}E_{||})$$

$$\mathbf{n}\cdot\mathbf{b} = 0 \Rightarrow \mathbf{E}_t = 0 \text{ or } k_t \to \infty$$

Future work



Singularity problem

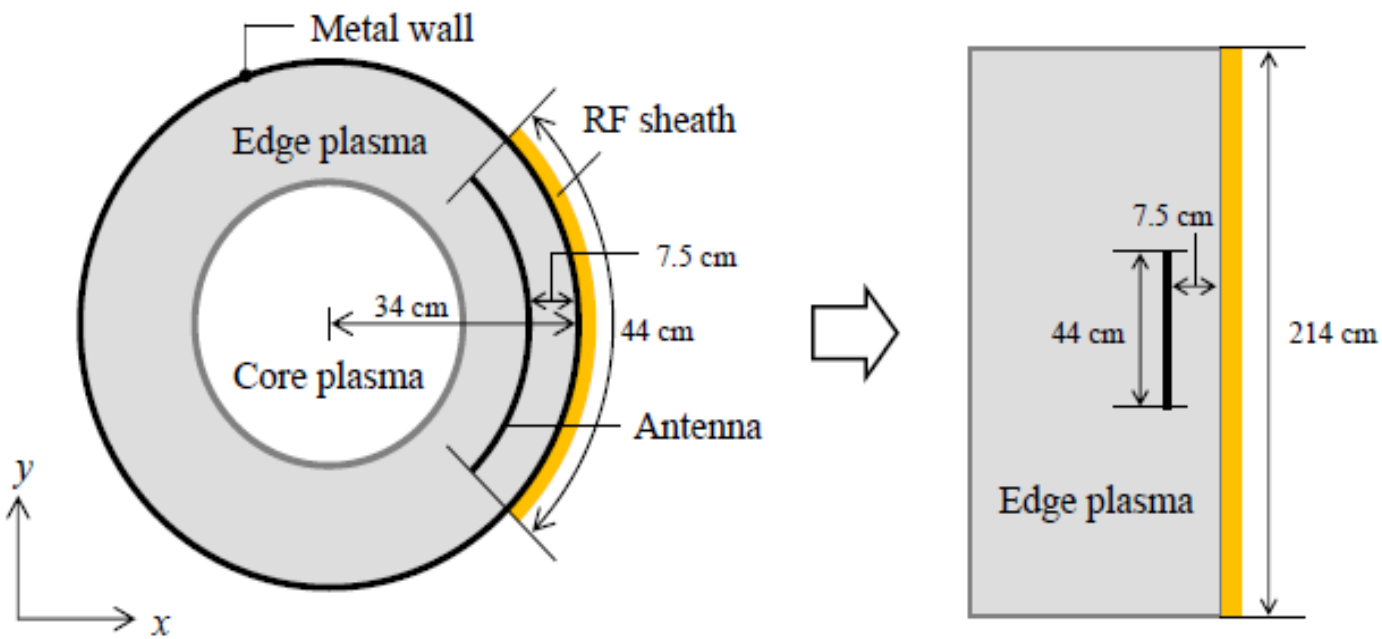
The wavenumber component tangential to the sheath surface rapidly increases to infinity when the magnetic field line approaches parallel to the wall.

Possible remedy:

- Modify the sheath BC so as to lessen the tangential wavenumber component at the singular points.
- Introduce a spectral approach for the discretization of the sheath BC.

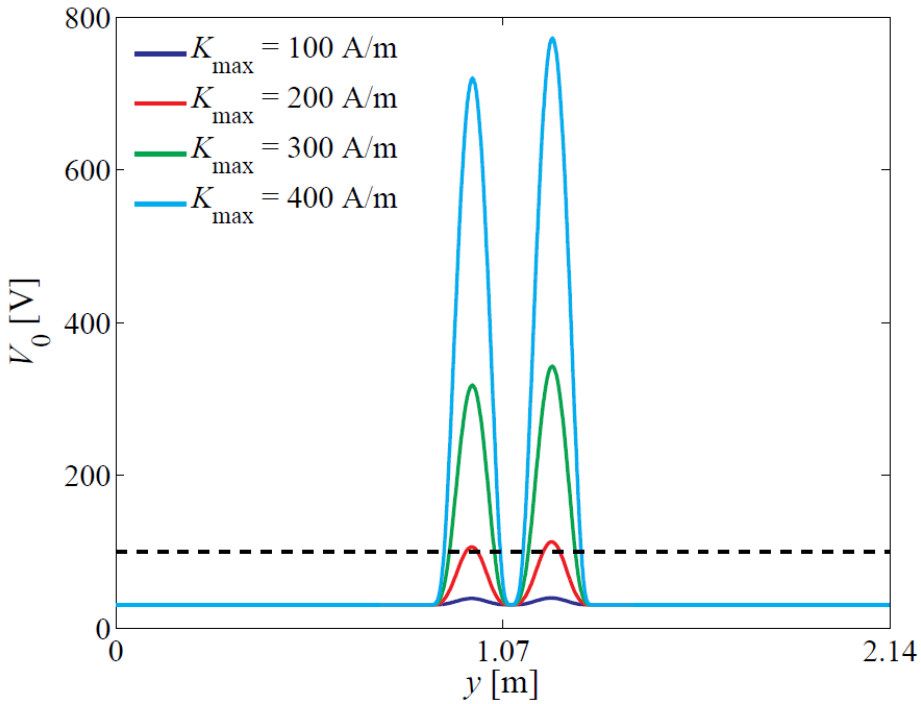
Preliminary application to Alcator C-Mod

Example 6: Numerical analysis for an Alcator C-Mod scale device



Dependence of sheath potential on antenna current for C-Mod model

$L_{\text{ant}} = 44$ cm



(also depends on poloidal length of antenna)

numerical details for rfSOL code

- finite element:

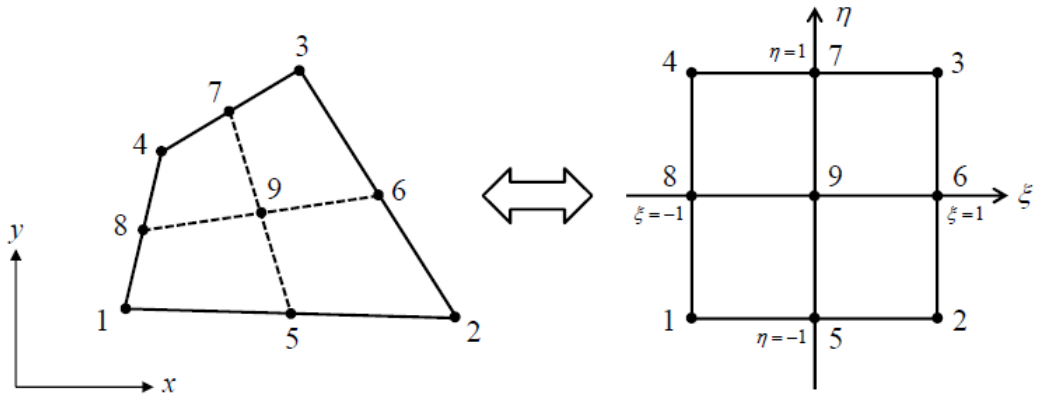
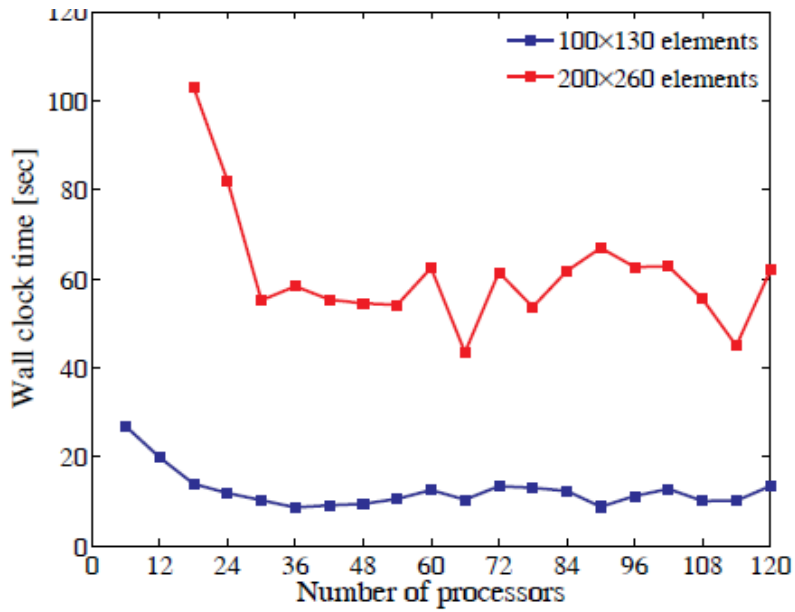


Figure 3-1: A nine-node element in the physical and mapping spaces.

- use MUMPS algorithm to solve sparse linear systems

run time at NERSC using MPI and Scalapack

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